

**KEC303      Network Analysis and Synthesis**

Unit 1: Node and mesh analysis, matrix approach of network containing voltage & current sources and reactances, source transformation and duality.

**Ohms Law Relationship**

$$\text{Current, (I)} = \frac{\text{Voltage, (V)}}{\text{Resistance, (R)}} \text{ in Amperes, (A)}$$

By knowing any two values of the Voltage, Current or Resistance quantities we can use **Ohms Law** to find the third missing value. **Ohms Law** is used extensively in electronics formulas and calculations so it is “very important to understand and accurately remember these formulas”.

**To find the Voltage, ( V )**

$$[ V = I \times R ] \quad V \text{ (volts)} = I \text{ (amps)} \times R \text{ (}\Omega\text{)}$$

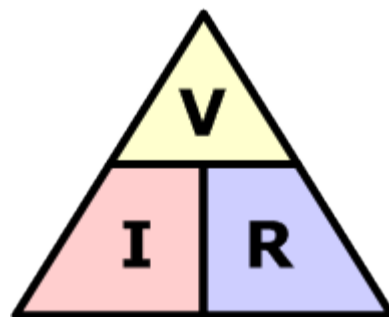
**To find the Current, ( I )**

$$[ I = V \div R ] \quad I \text{ (amps)} = V \text{ (volts)} \div R \text{ (}\Omega\text{)}$$

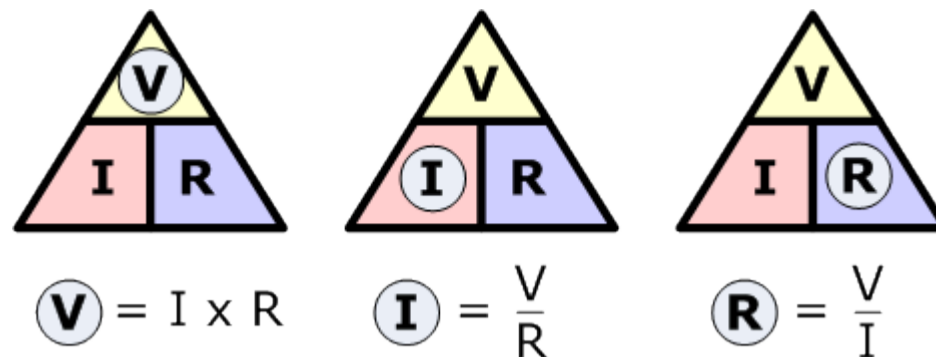
**To find the Resistance, ( R )**

$$[ R = V \div I ] \quad R \text{ (}\Omega\text{)} = V \text{ (volts)} \div I \text{ (amps)}$$

It is sometimes easier to remember this Ohms law relationship by using pictures. Here the three quantities of V, I and R have been superimposed into a triangle (affectionately called the **Ohms Law Triangle**) giving voltage at the top with current and resistance below. This arrangement represents the actual position of each quantity within the Ohms law formulas.

**Ohms Law Triangle**

Transposing the standard Ohms Law equation above will give us the following combinations of the same equation:



Then by using Ohms Law we can see that a voltage of 1V applied to a resistor of  $1\Omega$  will cause a current of 1A to flow and the greater the resistance value, the less current that will flow for a given applied voltage. Any Electrical device or component that obeys “Ohms Law” that is, the current flowing through it is proportional to the voltage across it ( $I \propto V$ ), such as resistors or cables, are said to be “**Ohmic**” in nature, and devices that do not, such as transistors or diodes, are said to be “**Non-ohmic**” devices.

### Electrical Power in Circuits

Electrical Power, ( P ) in a circuit is the rate at which energy is absorbed or produced within a circuit. A source of energy such as a voltage will produce or deliver power while the connected load absorbs it. Light bulbs and heaters for example, absorb electrical power and convert it into either heat, or light, or both. The higher their value or rating in watts the more electrical power they are likely to consume.

The quantity symbol for power is P and is the product of voltage multiplied by the current with the unit of measurement being the **Watt** ( W ). Prefixes are used to denote the various multiples or sub-multiples of a watt, such as: **milliwatts** ( $mW = 10^{-3}W$ ) or **kilowatts** ( $kW = 10^3W$ ).

Then by using Ohm’s law and substituting for the values of V, I and R the formula for electrical power can be found as:

### To find the Power (P)

$$[ P = V \times I ] \quad P \text{ (watts)} = V \text{ (volts)} \times I \text{ (amps)}$$

Also:

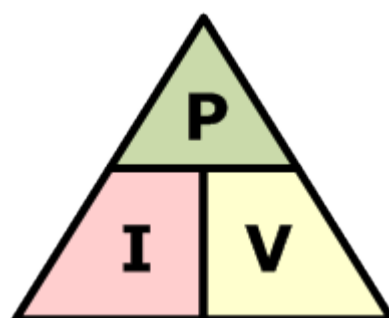
$$[ P = V^2 \div R ] \quad P \text{ (watts)} = V^2 \text{ (volts)} \div R \text{ (}\Omega\text{)}$$

Also:

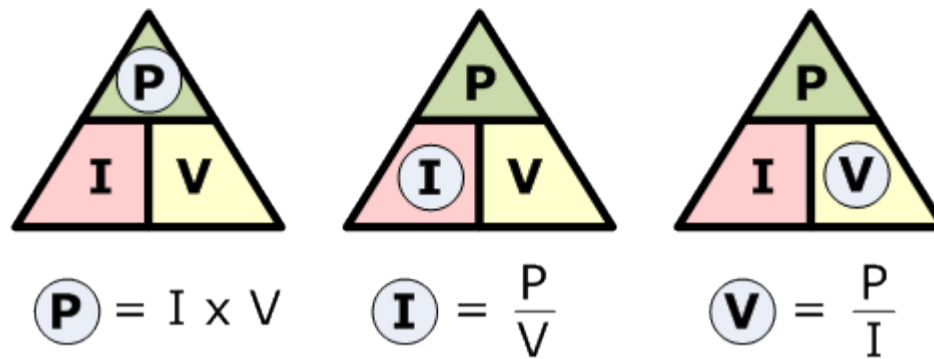
$$[ P = I^2 \times R ] \quad P \text{ (watts)} = I^2 \text{ (amps)} \times R \text{ (}\Omega\text{)}$$

Again, the three quantities have been superimposed into a triangle this time called a **Power Triangle** with power at the top and current and voltage at the bottom. Again, this arrangement represents the actual position of each quantity within the Ohms law power formulas.

### The Power Triangle



and again, transposing the basic Ohms Law equation above for power gives us the following combinations of the same equation to find the various individual quantities:

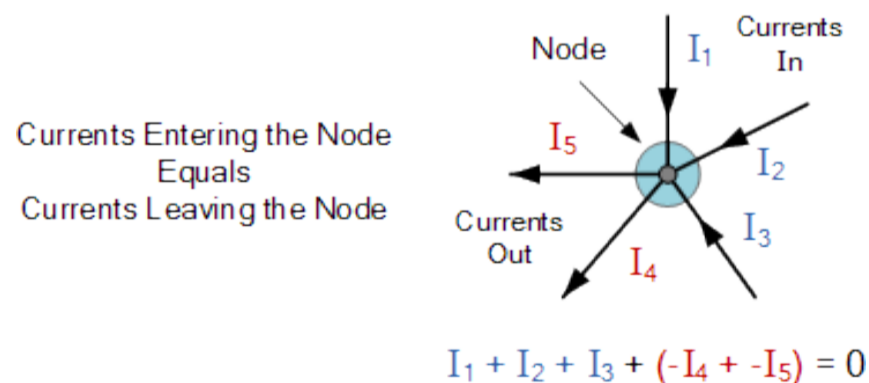


So we can see that there are three possible formulas for calculating electrical power in a circuit. If the calculated power is positive, (+P) in value for any formula the component absorbs the power, that is it is consuming or using power. But if the calculated power is negative, (-P) in value the component produces or generates power, in other words it is a source of electrical power such as batteries and generators.

### Kirchhoffs First Law – The Current Law, (KCL)

**Kirchhoffs Current Law** or KCL, states that the “total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node“. In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,  $I_{(\text{exiting})} + I_{(\text{entering})} = 0$ . This idea by Kirchhoff is commonly known as the **Conservation of Charge**.

### Kirchhoffs Current Law



Here, the three currents entering the node,  $I_1$ ,  $I_2$ ,  $I_3$  are all positive in value and the two currents leaving the node,  $I_4$  and  $I_5$  are negative in value. Then this means we can also rewrite the equation as;

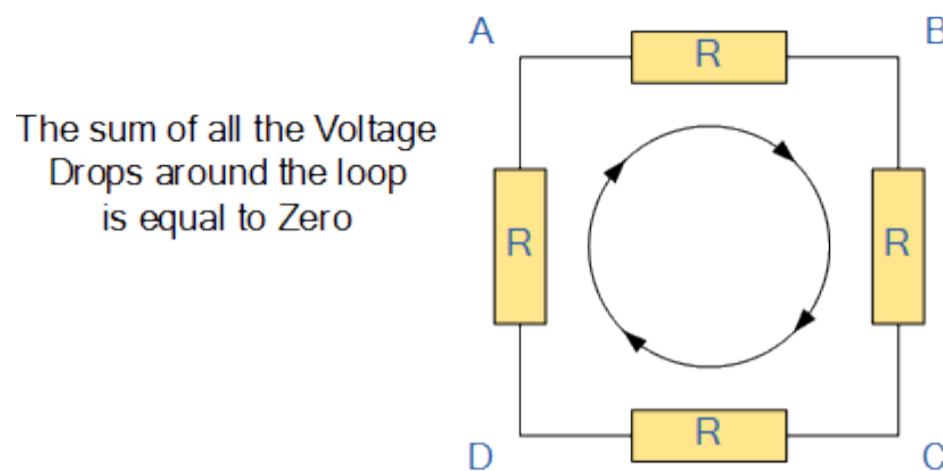
$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

The term **Node** in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchhoff's current law when analysing parallel circuits.

### Kirchhoffs Second Law – The Voltage Law, (KVL)

**Kirchhoffs Voltage Law** or KVL, states that “in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop” which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchhoff is known as the **Conservation of Energy**.

### Kirchhoffs Voltage Law



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Starting at any point in the loop continue in the **same direction** noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchhoff's voltage law when analysing series circuits.

When analysing either DC circuits or AC circuits using **Kirchhoffs Circuit Laws** a number of definitions and terminologies are used to describe the parts of the circuit being analysed such as: node, paths, branches, loops and meshes. These terms are used frequently in circuit analysis so it is important to understand them.

### Common DC Circuit Theory Terms:

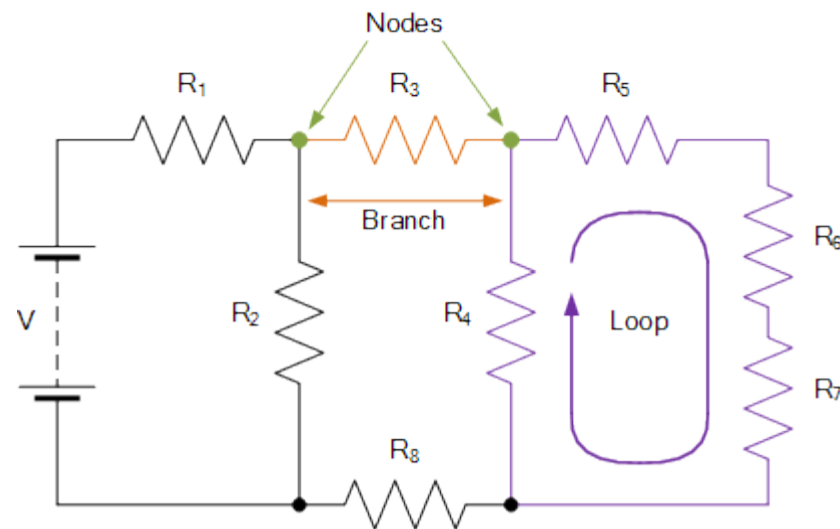
- Circuit – a circuit is a closed loop conducting path in which an electrical current flows.
- Path – a single line of connecting elements or sources.
- Node – a node is a junction, connection or terminal within a circuit where two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.
- Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.
- Loop – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once.
- Mesh – a mesh is a single closed loop series path that does not contain any other paths. There are no loops inside a mesh.

### Note that:

Components are said to be connected together in Series if the same current value flows through all the components.

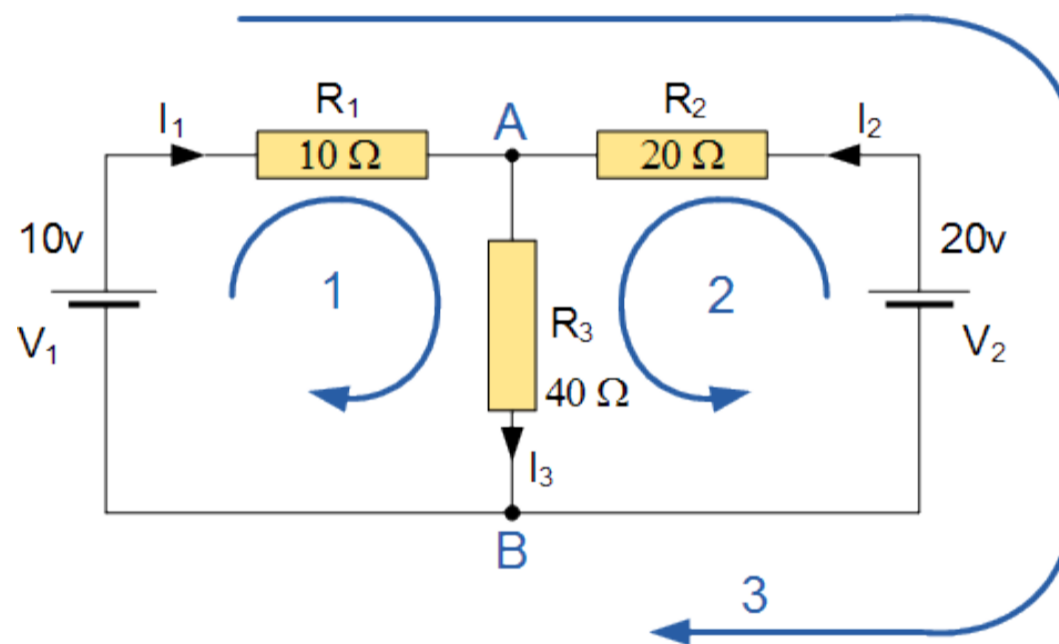
Components are said to be connected together in Parallel if they have the same voltage applied across them.

## A Typical DC Circuit



## Kirchhoff's Circuit Law Example No1

Find the current flowing in the  $40\Omega$  Resistor,  $R_3$



The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchhoff's Current Law, KCL** the equations are given as:

$$\text{At node A : } I_1 + I_2 = I_3$$

$$\text{At node B : } I_3 = I_1 + I_2$$

Using **Kirchhoff's Voltage Law, KVL** the equations are given as:

$$\text{Loop 1 is given as : } 10 = R_1 I_1 + R_3 I_3 = 10I_1 + 40I_3$$

$$\text{Loop 2 is given as : } 20 = R_2 I_2 + R_3 I_3 = 20I_2 + 40I_3$$

$$\text{Loop 3 is given as : } 10 - 20 = 10I_1 - 20I_2$$

As  $I_3$  is the sum of  $I_1 + I_2$  we can rewrite the equations as;

$$\text{Eq. No 1 : } 10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$$

$$\text{Eq. No 2 : } 20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$$

We now have two “**Simultaneous Equations**” that can be reduced to give us the values of  $I_1$  and  $I_2$

Substitution of  $I_1$  in terms of  $I_2$  gives us the value of  $I_1$  as  $-0.143$  Amps

Substitution of  $I_2$  in terms of  $I_1$  gives us the value of  $I_2$  as  $+0.429$  Amps

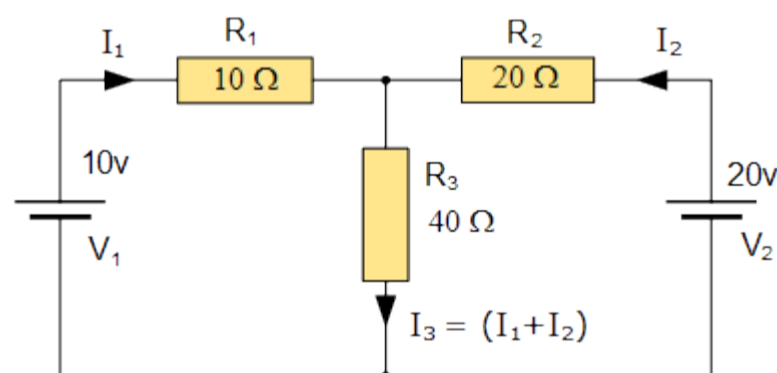
$$\text{As : } I_3 = I_1 + I_2$$

The current flowing in resistor  $R_3$  is given as :  $-0.143 + 0.429 = 0.286$  Amps

and the voltage across the resistor  $R_3$  is given as :  $0.286 \times 40 = 11.44$  volts

The negative sign for  $I_1$  means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

### Mesh Current Analysis Circuit



One simple method of reducing the amount of math's involved is to analyse the circuit using Kirchhoff's Current Law equations to determine the currents,  $I_1$  and  $I_2$  flowing in the two resistors. Then there is no need to calculate the current  $I_3$  as its just the sum of  $I_1$  and  $I_2$ . So Kirchhoff's second voltage law simply becomes:

- Equation No 1 :  $10 = 50I_1 + 40I_2$
- Equation No 2 :  $20 = 40I_1 + 60I_2$

therefore, one line of math's calculation have been saved.

### Mesh Current Analysis

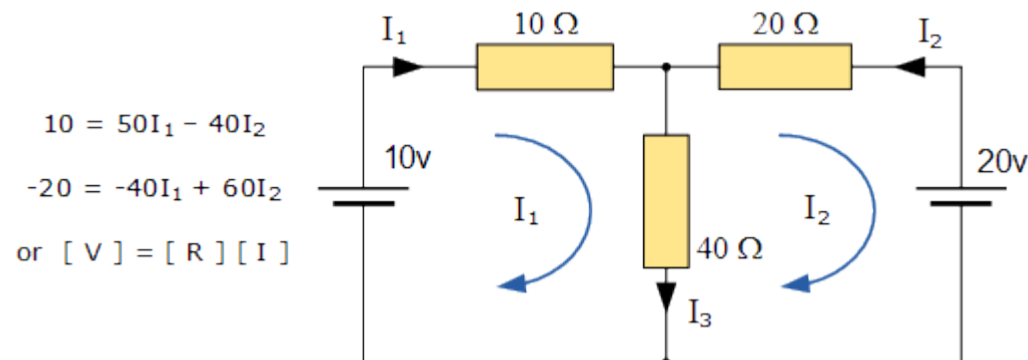
An easier method of solving the above circuit is by using **Mesh Current Analysis** or **Loop Analysis** which is also sometimes called **Maxwell's Circulating Currents** method. Instead of labelling the branch currents we need to label each “closed loop” with a circulating current.

As a general rule of thumb, only label inside loops in a clockwise direction with circulating currents as the aim is to cover all the elements of the circuit at least once. Any required branch current may be found from the appropriate loop or mesh currents as before using Kirchhoff's method.

For example: :  $i_1 = I_1$  ,  $i_2 = -I_2$  and  $I_3 = I_1 - I_2$

We now write Kirchhoff's voltage law equation in the same way as before to solve them but the advantage of this method is that it ensures that the information obtained from the circuit equations is the minimum required to solve the circuit as the information is more general and can easily be put into a matrix form.

For example, consider the circuit from the previous section.



These equations can be solved quite quickly by using a single mesh impedance matrix  $Z$ . Each element ON the principal diagonal will be “positive” and is the total impedance of each mesh. Where as, each element OFF the principal diagonal will either be “zero” or “negative” and represents the circuit element connecting all the appropriate meshes.

First we need to understand that when dealing with matrices, for the division of two matrices it is the same as multiplying one matrix by the inverse of the other as shown.

$$[V] = [I] \times [R] \quad \text{or} \quad [R] \times [I] = [V]$$

$$\begin{bmatrix} 50 & -40 \\ -40 & 60 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

$$I = \frac{V}{R} = R^{-1} \times V$$

$$\text{Inverse of } R = \begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix}$$

$$|R| = (60 \times 50) - (40 \times 40) = 1400$$

$$\therefore R^{-1} = \frac{1}{1400} \begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix}$$

having found the inverse of  $R$ , as  $V/R$  is the same as  $V \times R^{-1}$ , we can now use it to find the two circulating currents.



$$[I] = [R^{-1}] \times [V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{1400} \begin{bmatrix} 60 & 40 \\ 40 & 50 \end{bmatrix} \times \begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

$$I_1 = \frac{(60 \times 10) + (40 \times -20)}{1400} = \frac{-200}{1400} = -0.143 A$$

$$I_2 = \frac{(40 \times 10) + (50 \times -20)}{1400} = \frac{-600}{1400} = -0.429 A$$

Where:

- [ V ] gives the total battery voltage for loop 1 and then loop 2
- [ I ] states the names of the loop currents which we are trying to find
- [ R ] is the resistance matrix
- [ R<sup>-1</sup> ] is the inverse of the [ R ] matrix

and this gives I<sub>1</sub> as -0.143 Amps and I<sub>2</sub> as -0.429 Amps

As : I<sub>3</sub> = I<sub>1</sub> - I<sub>2</sub>

The combined current of I<sub>3</sub> is therefore given as : -0.143 - (-0.429) = 0.286 Amps

### Nodal Voltage Analysis

**Nodal Voltage Analysis** complements the previous mesh analysis in that it is equally powerful and based on the same concepts of matrix analysis. As its name implies, **Nodal Voltage Analysis** uses the “Nodal” equations of Kirchhoff’s first law to find the voltage potentials around the circuit.

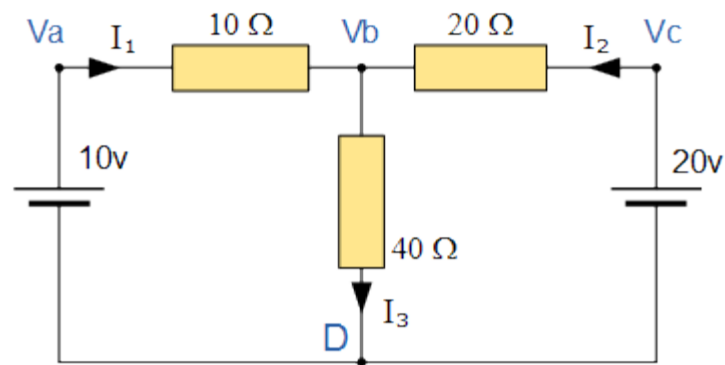
So by adding together all these nodal voltages the net result will be equal to zero. Then, if there are “n” nodes in the circuit there will be “n-1” independent nodal equations and these alone are sufficient to describe and hence solve the circuit.

At each node point write down Kirchhoff’s first law equation, that is: “*the currents entering a node are exactly equal in value to the currents leaving the node*” then express each current in terms of the voltage across the branch. For “n” nodes, one node will be used as the reference node and all the other voltages will be referenced or measured with respect to this common node.

For example, consider the circuit from the previous section.

### Nodal Voltage Analysis Circuit





In the above circuit, node D is chosen as the reference node and the other three nodes are assumed to have voltages,  $V_a$ ,  $V_b$  and  $V_c$  with respect to node D. For example;

$$\frac{(V_a - V_b)}{10} + \frac{(V_c - V_b)}{20} = \frac{V_b}{40}$$

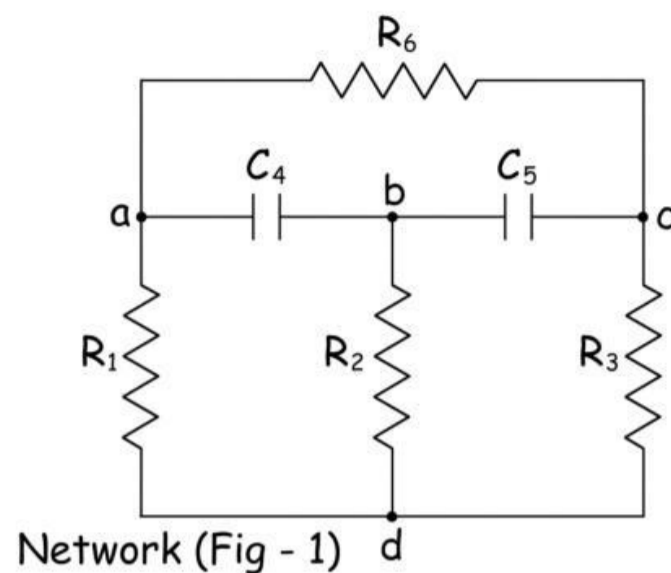
As  $V_a = 10\text{v}$  and  $V_c = 20\text{v}$ ,  $V_b$  can be easily found by:

$$\begin{aligned} \left(1 - \frac{V_b}{10}\right) + \left(1 - \frac{V_b}{20}\right) &= \frac{V_b}{40} \\ 2 &= V_b \left(\frac{1}{40} + \frac{1}{20} + \frac{1}{10}\right) \\ V_b &= \frac{80}{7} \text{ V} \\ \therefore I_3 &= \frac{2}{7} \text{ or } 0.286 \text{ Amps} \end{aligned}$$

again is the same value of 0.286 amps, we found using **Kirchhoff's Circuit Law** in the previous tutorial.

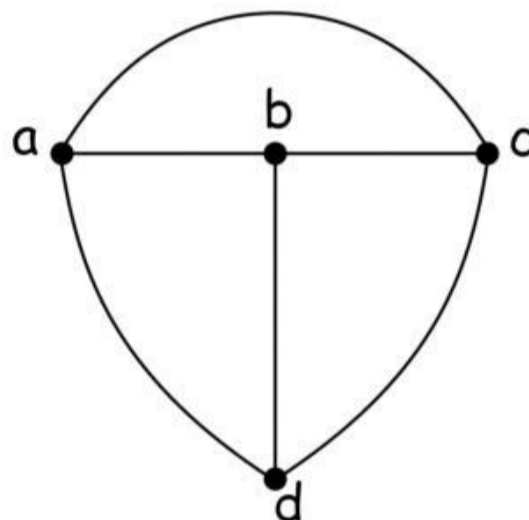
From both Mesh and Nodal Analysis methods we have looked at so far, this is the simplest method of solving this particular circuit. Generally, nodal voltage analysis is more appropriate when there are a larger number of current sources around. The network is then defined as:  $[I] = [Y][V]$  where  $[I]$  are the driving current sources,  $[V]$  are the nodal voltages to be found and  $[Y]$  is the admittance matrix of the network which operates on  $[V]$  to give  $[I]$ .

**Network Analysis** is a process by which we can calculate different electrical parameters of a circuit element connected in an electrical network. An electrical circuit or network can be complicated too and in a complicated network, we have to apply different methods to simplify the network for determining the electrical parameters. The circuit elements in a network can be connected in different manners, some of them are in series and some of them in parallel. The circuit elements are resistors, capacitors, inductors, voltage sources, current sources etc. Current, voltage, resistance, impedance, reactance, inductance, capacitance, frequency, electric power, electrical energy etc are the different electrical parameters we determine by **network analysis**. In short, we can say, an electrical network is the combination of different circuit elements and the network analysis or circuit analysis is the technique to determine the different electrical parameters of those circuit elements.



### Graph of an Electrical Network

When we replace all the circuit elements of an electrical network by hand-drawn lines, then the figure is known as the **graph of the network**. The figure – 2 below shows the **graph** of the above network in figure – 1.



The line representing the circuit element is called the branch of a network. The point where two or more branches meet is called a node of the network. The direction of current through the element is represented by an arrowhead drawn on the branch. The direction of the current in a graph can be considered arbitrarily. When we draw a graph of a network with the direction of current (the direction may be arbitrary) in each of the branches, the graph is called **oriented graph** of the network. The figure – 3 below shows the **oriented graph** of the above network in figure – 1.

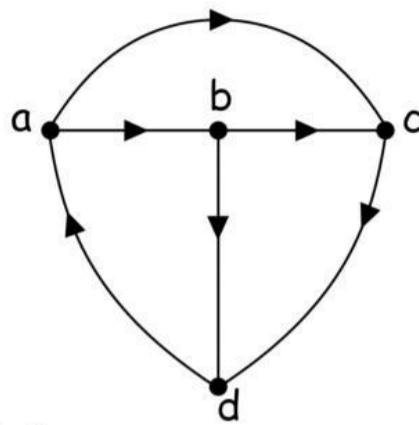


Fig 3

When an active network is represented as a passive network through a graph by removing the voltage and current sources then the graph is known as the **oriented topological graph** of the network. The voltage source is removed by replacing it with a short circuit and the current source is removed by replacing it with an open circuit.

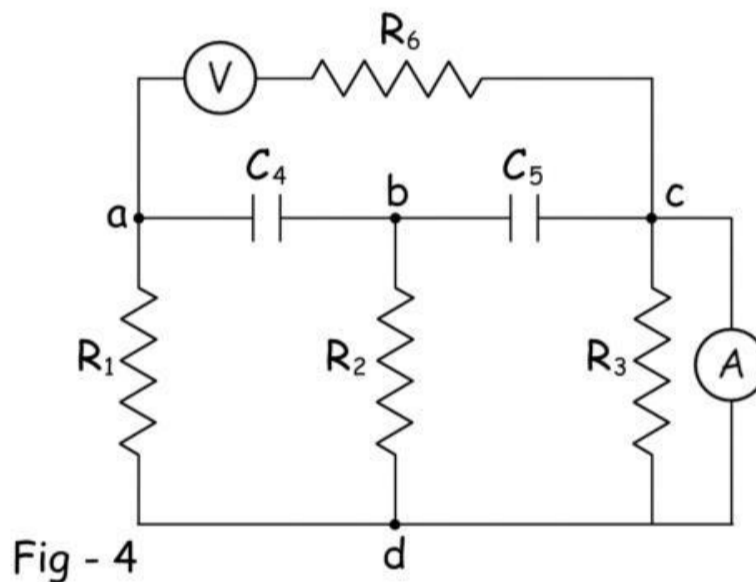


Fig - 4

The above figure – 4 shown an electrical network with both voltage source and current source. The figure – 5 below shows the **oriented topological graph** of the network in figure – 4.

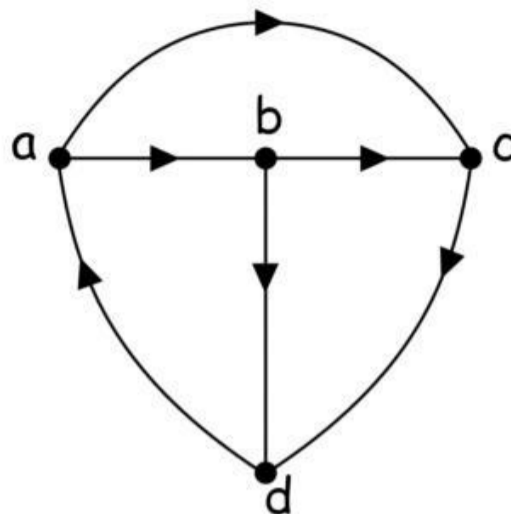


Fig 5

### Definition of Terms used in Network Analysis

#### Branch

Each hand drawn line in a graph which represents the path for flowing of current is called branch.

#### Node

The end point of the branch where other branches meet is called a node.

#### Subgraph

This is a subset of branches of a graph.

#### Tree

The tree is a subgraph which contains all nodes of the graph but does not form any closed circuit. If the graph has  $n$  number of nodes, the tree will have  $(n - 1)$  number of branches. The branches of a tree are referred to as twigs. Hence a tree can also be referred to as a set of twigs.

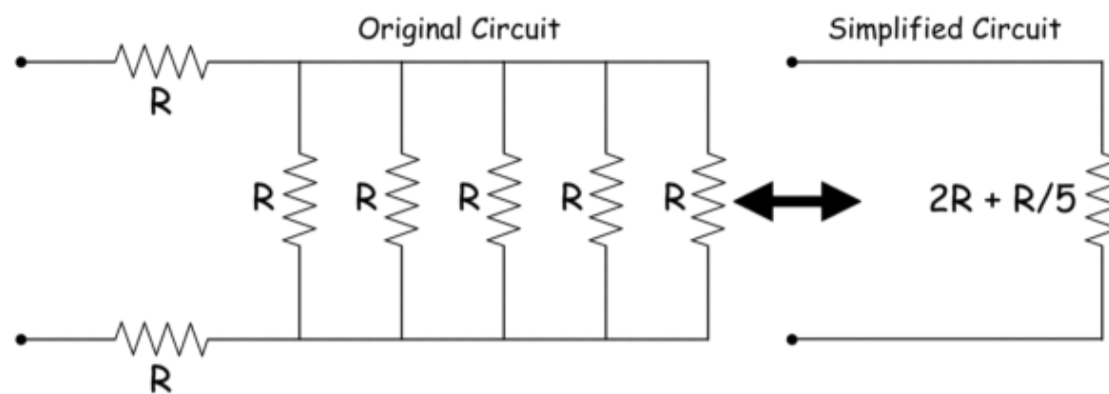
#### Cotree

The cotree is a subgraph which contains all those branches which are not included in a tree. The cotree is the complement of a tree.

#### Equivalent Circuit

The main step of network analysis is to simplify comparatively complex network to its simplified form. It may normally be done by combining impedances in series and parallel. Sometimes it requires to transform some or all of the voltage sources of the network to current source and vice versa. If we consider any two terminals of an active network, obviously there would be a voltage across the terminals and current through the terminals. After simplification of the network, across these two terminals, the voltage and current in respect of the terminals remain unaltered from the original one. Although the structure of the network has been changed significantly. The original circuit (or network) and the simplified circuit (or network) are called equivalent circuit of each other. In case of a passive network, the impedance across any two reference terminal of the network remains same after simplifying the

network.



### Series and Parallel Circuit

During network analysis most frequently done activities are combining series and parallel circuit elements. If  $n$  number of resistances are connected in series, the value of equivalent resistance would be,

$$R_e = R_1 + R_2 + R_3 + \cdots + R_n$$

If  $n$  number of resistances are connected in parallel, the value of equivalent resistance would be,

$$R_e = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \right)^{-1}$$

If  $n$  number of inductances are connected in series, the value of equivalent inductance would be,

$$L_e = L_1 + L_2 + L_3 + \cdots + L_n$$

If  $n$  number of inductances are connected in parallel, the value of equivalent inductance would be,

$$L_e = \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} \right)^{-1}$$

If  $n$  number of capacitances are connected in series, the value of equivalent capacitance would be,

$$C_e = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \right)^{-1}$$

If  $n$  number of capacitances are connected in parallel, the value of equivalent capacitance would be,

$$C_e = C_1 + C_2 + C_3 + \cdots + C_n$$

If  $n$  number of impedances are connected in series, the value of equivalent impedance would be,

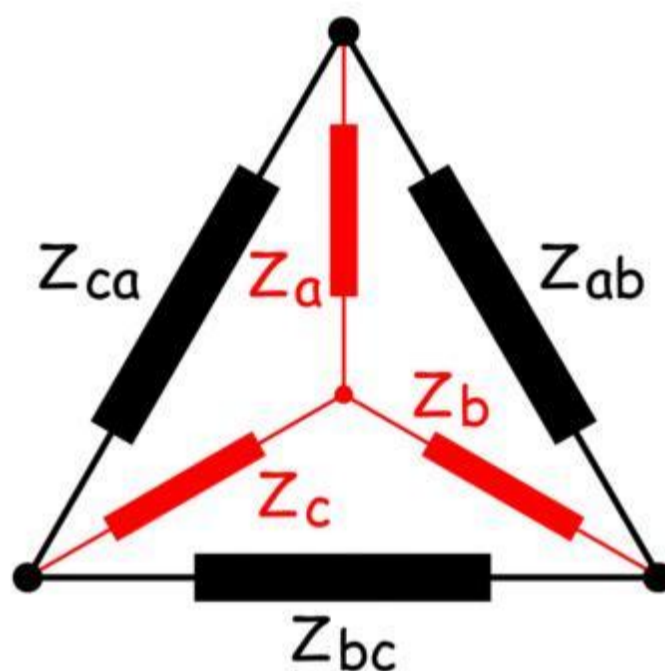
$$Z_e = Z_1 + Z_2 + Z_3 + \cdots + Z_n$$

If n number of impedances are connected in parallel, the value of equivalent impedance would be,

$$Z_e = \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots + \frac{1}{Z_n} \right)^{-1}$$

### Star Delta Transformation

In case of two terminal network the impedances between the ports can easily be simplified by the series-parallel combination of the impedances. If the number of terminals in a network is more than two then the equivalent impedance between the terminals may not be solved by simple series and parallel combination of the impedances. Let us consider three terminal network. The impedances or any other similar parameters between the ports are either star connected or delta connected. The delta connected network can be converted into a star connected network and vice versa. During network analysis, we have to transform either delta to star or star to delta for simplifying the network. Let us consider one three terminal network formed by three impedances  $Z_a$ ,  $Z_b$ , and  $Z_c$  connected in star. Consider another three terminal network formed by three impedances  $Z_{ab}$ ,  $Z_{bc}$ , and  $Z_{ca}$ . If these two networks are equivalent to each other then the relationship between the impedances of star and delta would be as follows.



$$Z_a = \frac{Z_{ca}Z_{ab}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_b = \frac{Z_{ab}Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_c = \frac{Z_{bc}Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

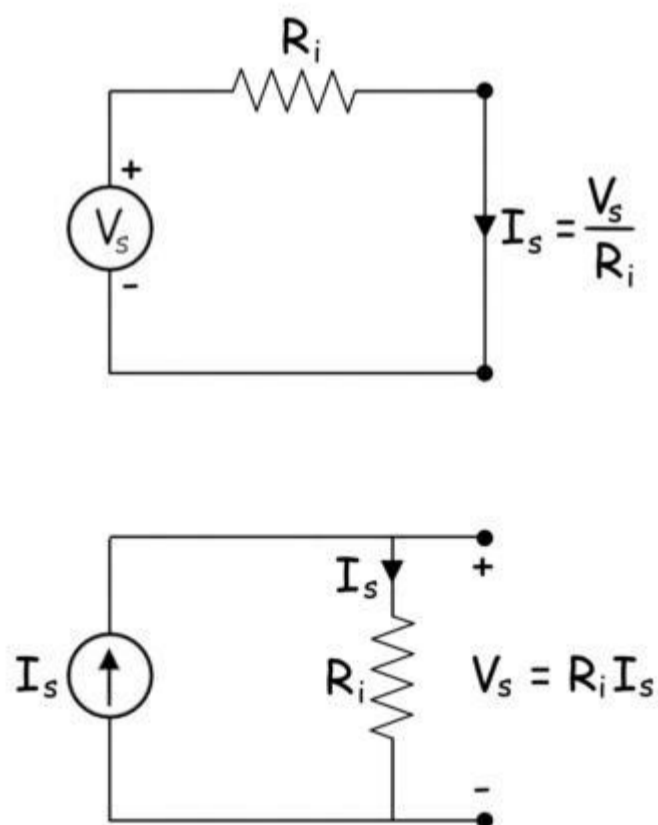
$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$

$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

### Electrical Source Transformation

Another essential step very often used during network analysis is source transformation. Often it becomes essential to convert current sources to voltage sources and voltage sources to current sources for simplification of complex electrical network. In reality a practical voltage source can be considered as an ideal voltage source in series with its internal resistance. In the same way, a practical current source can be considered as an ideal current source in parallel with its internal resistance. When a voltage source is connected with a circuit it imposes its the voltage across the terminals of the circuit and it delivers certain current to the circuit depending on the impedance of the circuit and series internal resistance of the source. A current source can be said equivalent of the voltage source when the current source delivers the same current to the circuit when connected across same terminals. It is found that the current of the current source would be short circuit current of the voltage source and the value of internal resistance is same as that of voltage source but connected in parallel instead of series. That means if we short two terminals of a voltage source, the current flowing through the device is the current of the equivalent current source. Similarly, when a current source is open circuited, the voltage appears across the open terminal of the source would be the voltage of the equivalent voltage source.

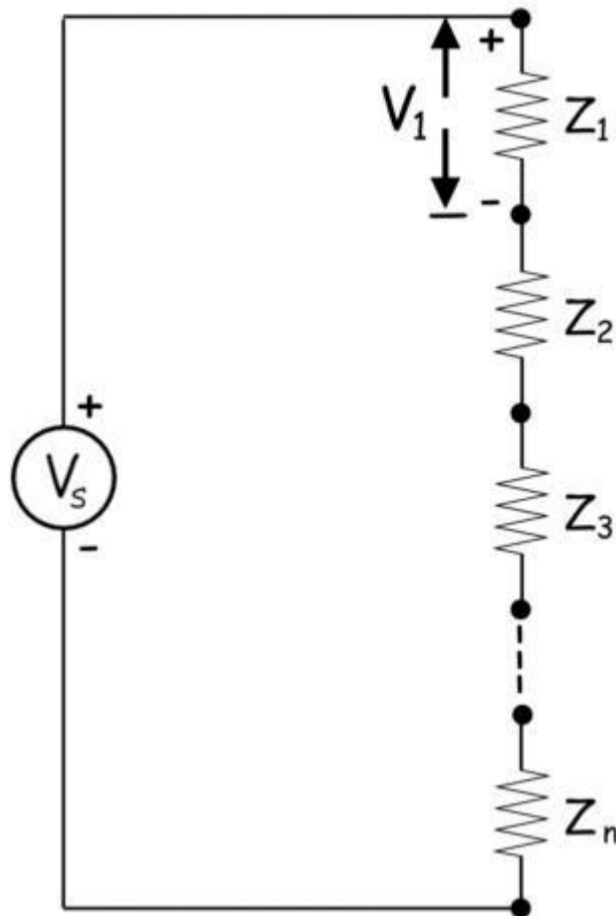


### Voltage Current Division Rule

The often used techniques for network analysis are voltage and current division rule. Voltage division rule the process for calculating the voltage drop across a particular impedance among a series of impedances across a voltage source.



Suppose, there are  $n$  number of impedances  $Z_1, Z_2, Z_3, \dots, Z_n$  connected in series across a voltage source of voltage  $V_s$ . Then the voltage drop across the impedance  $Z_1$  is,



Here in the figure above we have considered impedances as resistances but we can consider any other impedance parameters instead of resistance according to the requirement of the circuit.

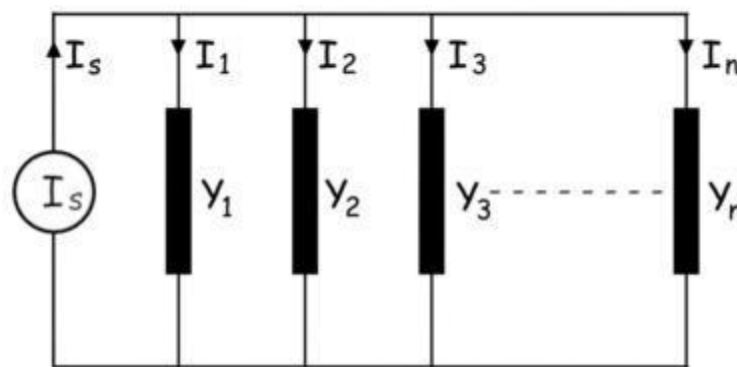
$$V_1 = \frac{V_s Z_1}{Z_1 + Z_2 + Z_3 + \dots + Z_n}$$

Similarly, the voltage drop across any particular impedance  $Z_i$  is given by

$$V_i = \frac{V_s Z_i}{Z_1 + Z_2 + Z_3 + \dots + Z_n}$$

Where,  $i = 1, 2, 3, \dots, n$

Let us take one current source of current  $I_s$  connected across  $n$  number of parallel admittance  $Y_1, Y_2, Y_3, \dots, Y_n$ .



The current passing through the admittance  $Y_1$  is expressed as

$$I_1 = \frac{I_s Y_1}{Y_1 + Y_2 + Y_3 + \dots + Y_n}$$

Similarly, the current passing through the admittance  $Y_i$  is,

$$I_i = \frac{I_s Y_i}{Y_1 + Y_2 + Y_3 + \dots + Y_n}$$

Where,  $i = 1, 2, 3, \dots, n$

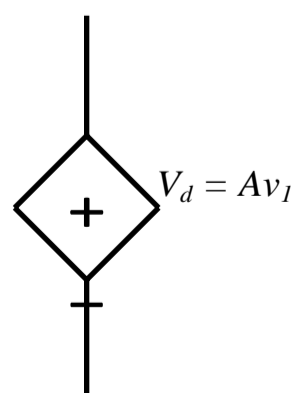
### Dependent sources

As we begin to use simple circuits to model more complex circuit behavior, we need to add some items to our tool kit. Dependent sources behave just like independent voltage and current sources, except that the voltage or current depends in some way on another voltage or current in the circuit.

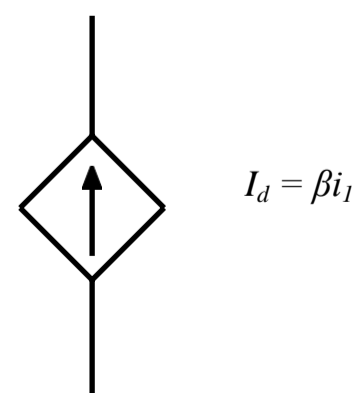
This seems a bit odd, but this behavior corresponds very closely to the way a number of interesting and useful electronic devices behave.

We're not to try to get a detailed understanding of how these devices work internally – that's the subject for an electronics or semiconductor class. However, we can form a reasonable *model* of how the electronic devices behaves in a circuit by using dependent sources.

### Dependent voltage source



### Dependent current source



Here  $v_l$  and  $i_l$  are quantities defined somewhere else in the circuit, including the proper polarity or direction. These definitions must be included, or the circuit is not properly specified.

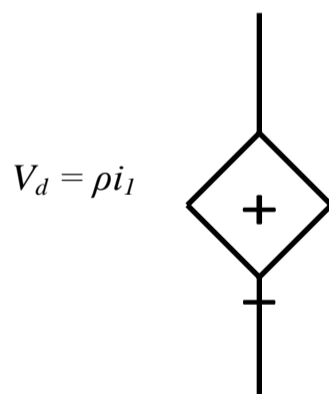
Note that the dependency factors  $A$  and  $\beta$  are dimensionless quantities.

For the voltage source above, since the voltage depends on another voltage, it is known as a *voltage-controlled voltage source (VCVS)*.

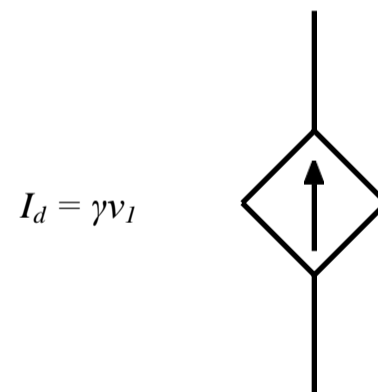
Similarly, the current would be called a *current-controlled current source (CCCS)*.

It is not necessary that the voltage source be dependent on another voltage or that the current source depend on another current.

Current-controlled voltage source  
(CCVS)



voltage-controlled current source  
(VCCS)



Again, the controlling current,  $i_l$  and the controlling voltage  $v_l$  must be defined somewhere else in the circuit.

In these cases, the dependency factors will have units. For  $\rho$ , the units are  $\Omega$ . This does not mean that  $\rho$  represents some type of resistor – it is simply the factor that relates the voltage to its controlling current.

The units for  $\gamma$  must be siemens ( $S = A/V = \Omega^{-1}$ ).

Once the dependent source are located in circuit, along with the definitions for the controlling currents or voltages, then circuit analysis proceeds as always. Kirchoff's current and voltage laws still apply and all of the techniques derived from those still apply. In particular, voltage dividers, the node-voltage method, and the

loop current technique are unchanged. Source transformations must be used with caution. Since the dependent source is defined in terms of a particular voltage or current, you must be careful about changing the definitions – the overall circuit behavior must remain unchanged.

When using superposition, dependent sources *cannot* be removed. The dependent source must stay in place for all of the partial circuits you consider each *independent* source in turn.

When doing Thevenin equivalents, you cannot remove the dependent sources when trying to determine the equivalent resistance using the *short-cut* method. Thus, when dependent sources are present, the short-cut technique becomes somewhat less useful.

As long as you remember those caveats for the source transformations, superposition, and Thevenin equivalents, everything that we've learned to this point can be applied to circuits with dependent sources.